Lifetime scaling in non-neutral plasmas

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(September 17, 1998)

Abstract

The non-neutral plasma community has long believed that the lifetime of plasmas held in Penning-Malmberg traps scales as $\tau_{\mathrm{m}} \propto B^2/L^2$. The lifetime, τ_{m}, is the time for the plasma's central density to decrease by two, B is the trap's axial magnetic field, and L is the plasma length. However, the case for this scaling law is weak. Other scaling laws work just as well. In particular, $\tau_{\mathrm{m}} \propto B^2/L^3$ is in equally good agreement with lifetime measurements. This scaling would result from transport induced by quadrupolar magnetic field errors.

Key Words: *non-neutral plasma lifetimes, resonant particle transport, quadrupolar fields errors*

The plasma expansion mechanism in Penning-Malmberg non-neutral plasma traps is not understood [1]. Two decades of research has yielded little more than an empirical lifetime scaling law and the general observation that external asymmetries usually, though not always, decrease the plasma lifetime [2].

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The generally accepted lifetime scaling law is [3,4]

$$\tau_{\mathrm{m}} \propto \frac{B^2}{L^2}, \tag{1}$$

where τ_{m} is the time (in seconds) for the plasma's central density to decrease by two, B is the trap's axial magnetic field, and L is the plasma length. This scaling is roughly correct in many different devices, but is principally based on lifetime data taken by Driscoll *et al.* in two traps at the University of California at San Diego (UCSD) called the V trap [3] and the V$'$ trap [4]. For reference, the UCSD data is reproduced in Figs. 2a and 3a. Lifetime measurements taken at fifty five permutations of five different magnetic fields and eight plasma lengths in the V trap are shown in Fig. 2, while lifetime measurements taken at twenty seven permutations of six magnetic fields and five plasma lengths in the V$'$ trap are shown in Fig. 3. (Some points are repeated.) Also shown are the best-fit lines through the data, $\tau_{\mathrm{m}} = 0.0146 B^2/L^2$ (V), and $\tau_{\mathrm{m}} = 0.328 B^2/L^2$ (V$'$) [5]. The best-fit lines are based on minimizing the error,

$$E = \exp\left[\sqrt{\frac{1}{N-1}\sum_N \left\{\ln\left[\frac{\tau_{\mathrm{m}}(\mathrm{exp})}{\tau_{\mathrm{m}}(\mathrm{calc})}\right]\right\}^2}\right], \tag{2}$$

for each data set. Here $\tau_{\mathrm{m}}(\mathrm{exp})$ is the measured lifetime, $\tau_{\mathrm{m}}(\mathrm{calc})$ is the calculated lifetime from the scaling law, and the sum is over the N points in the dataset. Lifetime data is difficult to obtain accurately and is subject to pesky systematic errors, so it is not surprising that the fits are imperfect.

B^2/L^2 scaling is consistent with a simplistic model of resonant particle transport. Resonant particle transport is caused by the resonant interaction of a particle with a field error. Consider the motion of an individual plasma particle: the particle bounces from one end of the trap to the other with a characteristic transit frequency $\omega_{\mathrm{T}} = 2\pi v/L$, where v is the particle's velocity. Because the plasma is charged, it has a strong radial self-electric field, and the resultant $\mathbf{E} \times \mathbf{B}$ cause the particle to circle the plasma with a characteristic frequency $\omega_{\mathrm{R}} \propto n/B$, where n is the plasma density. Now assume that unwanted field errors kick the particle every time it reaches one end. If the kicks are always in the same transverse direction, say in $\hat{x}$, they will tend to increase the particle's distance from the plasma center when

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the particle is on one side of the plasma (positive x), and decrease the particle's distance from the plasma center when the particle is on the other side (negative x). In general, the transport is weak as the kicks tend to cancel out. But if the ratio ω_R/ω_T is such that the particle is always on the same side of the plasma when it receives the kick, the particle's excursion will be large and there will be strong transport.

The resonant ratio ω_R/ω_T is proportional to B/L. As most researchers blame resonant particle transport for the imperfect confinement in Penning-Malmberg traps (without, I should add, significant experimental evidence), the coincidence between the empirical lifetime scaling B^2/L^2 and the resonant ratio B/L has been too suggestive to ignore, and B^2/L^2 scaling has been generally accepted as fact. Unfortunately, this reasoning has a nagging defect. One would expect to see some signature of exact resonances. Transport should be higher when the plasma rotation frequency ω_R and transit frequency ω_T are commensurate. Failing this, one should at least see differences in transport above and below the principle resonance $\omega_T/\omega_R = 2$. But no such differences have been observed.

Worse, closer examination of the data shows that the experimental evidence for B^2/L^2 scaling is unconvincing. Other scalings work just as well, if not better. If noninteger exponents are allowed, then the best-fit lines are $0.047B^{1.979}/L^{2.364}$ (V) and $0.955B^{1.979}/L^{2.364}$ (V$'$) The fit errors (Eq. 2) are 2.5 (V) and 2.4 (V$'$). These are slightly better than the fit errors 2.6 (V) and 2.6 (V$'$) for B^2/L^2 scaling.

Another possible scaling is $\tau_m \propto B^2/L^3$. This scaling is shown in Figs. 2(b) and 3(b). The best-fit lines are $0.267B^2/L^3$ (V) and $4.67B^2/L^3$ (V$'$), and the fit errors are 3.2 (V) and 2.7 (V$'$), slightly worse then the B^2/L^2 fit errors. If the fit and error analysis is restricted to the 34 points in Fig. 2 and the 15 points in Fig. 3 for which $B/L > 6.7\,\mathrm{G/cm}$, then the B^2/L^3 fit errors are 2.6 (V) and 2.2 (V$'$), which are somewhat better than the similarly restricted B^2/L^2 fit errors 2.9 (V) and 2.8 (V$'$). A plausible justification for this restriction is given later.

B^2/L^3 scaling is particularly interesting, as it results from resonant particle transport induced by a quadrupolar magnetic field. Assume that there is a quadrupolar magnetic

field $\mathbf{B} = B_q(r/R_W)\left(\hat{r}\cos 2\theta + \hat{\theta}\sin 2\theta\right)$, where R_W is the trap wall radius. Particles whose velocity is such that they travel from one end of the trap to the other while making a quarter rotation follow magnetic field lines which take the particles ever outwards or inwards. This class of resonant particles is supplemented by other, less important classes of resonant particles that travel from end to end in the time it takes to make an odd integer times a quarter rotation. Preliminary experiments with deliberately-applied quadrupolar field errors demonstrate enhanced transport at the resonance [6].

A lifetime scaling law for quadrupole induced transport can be derived from the diffusion constant

$$D = \sum_{N\,\mathrm{odd}} \lambda_N^2 \nu f_N, \tag{3}$$

where λ_N is the step size of the Nth resonance, ν is the collision frequency, and f_N is the fraction of the plasma participating in the Nth resonance. Then the plasma lifetime τ is approximately r_p^2/D, where r_p is the radius of the plasma. The derivation will be given elsewhere [6]. The result is

$$\tau = C\frac{B^2}{B_q^2}\frac{B^2}{L^3}\left[\sum_{N\,\mathrm{odd}} \frac{1}{N^5}\exp\left(-\frac{v_N^2}{2v_T^2}\right)\right]^{-1}. \tag{4}$$

Here $C = \pi^{4.5}\epsilon_0^2 R_W^2 v_T/2^{2.5}e^2 n^2$, e and m are the electron charge and mass, v_T is the thermal velocity, and v_N is the Nth resonant velocity. The numerical factors in the constant C should not be taken too seriously. Note that two powers of L in Eq. (4) come from the step size λ, while one power of L comes from the resonant fraction f. If the quadrupole field results from asymmetries in the main field, $B_q \propto B$, and two powers of B in Eq. (4) are canceled. The argument of the exponent in Eq. (4) is

$$\frac{v_N^2}{2v_T^2} = \frac{e^2 n^2 m}{2\pi^2 \epsilon_0^2 kT}\frac{1}{N^2}\frac{L^2}{B^2}. \tag{5}$$

When B/L is large, this argument is small, and the sum in Eq. (4) approaches unity. As B/L decreases, the sum diminishes. Thus the lifetime scaling is B^2/L^3 when B/L is sufficiently large.

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Figure 4 compares Driscoll *et al.*'s data to the prediction of Eq. 4. The plasma temperature and density are needed to make this comparison, but are not well specified in the Driscoll *et al.*'s paper. I use $n = 6 \times 10^6 \, \text{cm}^{-3}$ and $T = 4.3 \, \text{eV}$ [7]. The value of C was found by fitting it to minimize the error [Eq. (2)].

The predicted lifetimes are in good agreement with the measured data for large B/L. Somewhere near $B/L \approx 2 \, \text{G/cm}$ the principle resonant velocity becomes larger than the thermal velocity, quadrupolar induced transport diminishes, and the calculated lifetime diverges from the measured lifetime. The exact point at which this occurs is very sensitive to the density and temperature, and cannot be determined precisely given the imprecision with which these numbers are reported. If quadrupolar induced transport is responsible for transport at large B/L, some other mechanism must be responsible for transport at small B/L.

From the fitted scaling coefficients C, one can infer the required magnetic field error. Since the formula for C is imprecise, the inferred error field is only an estimate. Nonetheless, the required quadrupolar field errors, $6.6 \times 10^{-4} rB/R_{\text{W}}$ (V) and $1.4 \times 10^{-4} rB/R_{\text{W}}$ (V′) are quite plausible.

In conclusion, I have shown that the case for the established scaling law, B^2/L^2 is weak. The data are compatible with the B^2/L^3 scaling that results from quadrupole magnetic field errors. Does the data prove that this scaling is correct? Certainly not! But the case for B^2/L^3 scaling is equally as good as the case for B^2/L^2 scaling.

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[7] These values take into account the plasma expansion.
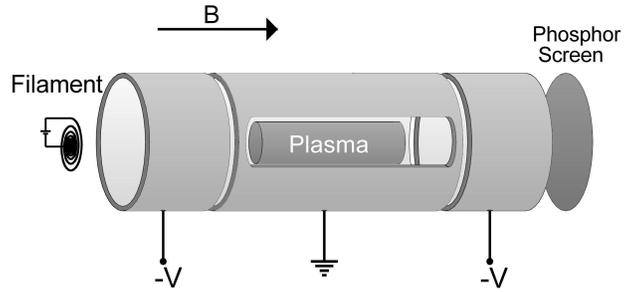
FIGURES

FIG. 1. A simplified schematic drawing of a Penning-Malmberg electron trap. Longitudinal confinement is provided by appropriately biasing the cylinders. Radial confinement is provided by the axial magnetic field **B**. The pure-electron plasma is generated by thermionic emission from the hot tungsten filament on the left, and loaded into the trap by momentarily grounding the leftmost cylinder. The plasma is imaged by momentarily grounding the rightmost cylinder, thereby allowing the plasma to stream onto the phosphor screen.
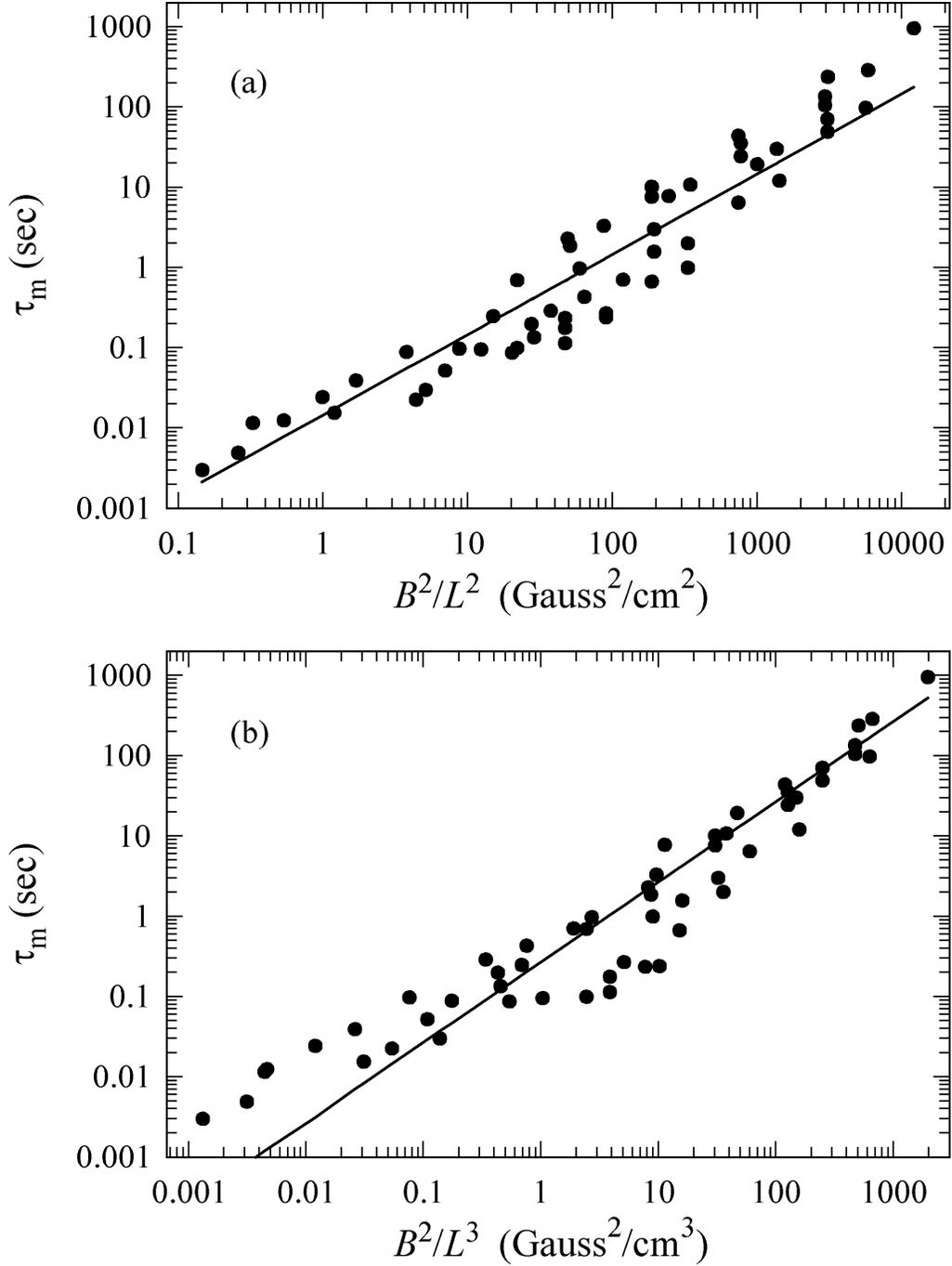
FIG. 2. (a) Reproduction of Driscoll *et al.*'s data for the V trap. (b) The same data plotted against L^3/B^2 instead of L^2/B^2. In both graphs the ● are the lifetime measurements, and the lines are the best-fit scaling law. The deviation in (b) at small B^2/L^3 of the B^2/L^3 line from the data might be a signature of resonant particle transport.
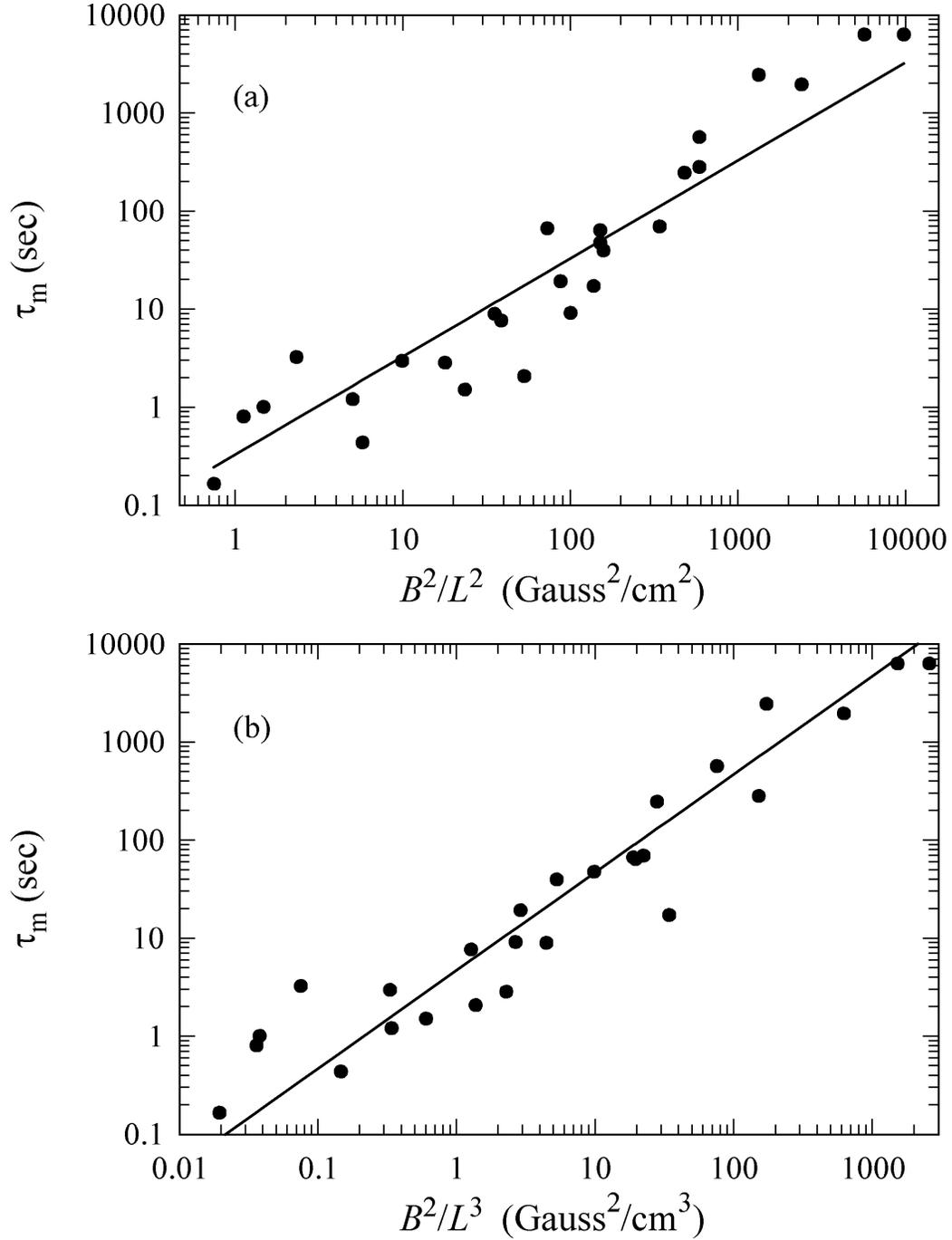
FIG. 3. (a) Reproduction of Driscoll *et al.*'s data for the V′ trap. (b) The same data plotted against L^3/B^2 instead of L^2/B^2. In both graphs the ● are the lifetime measurements, and the lines are the best-fit scaling law.
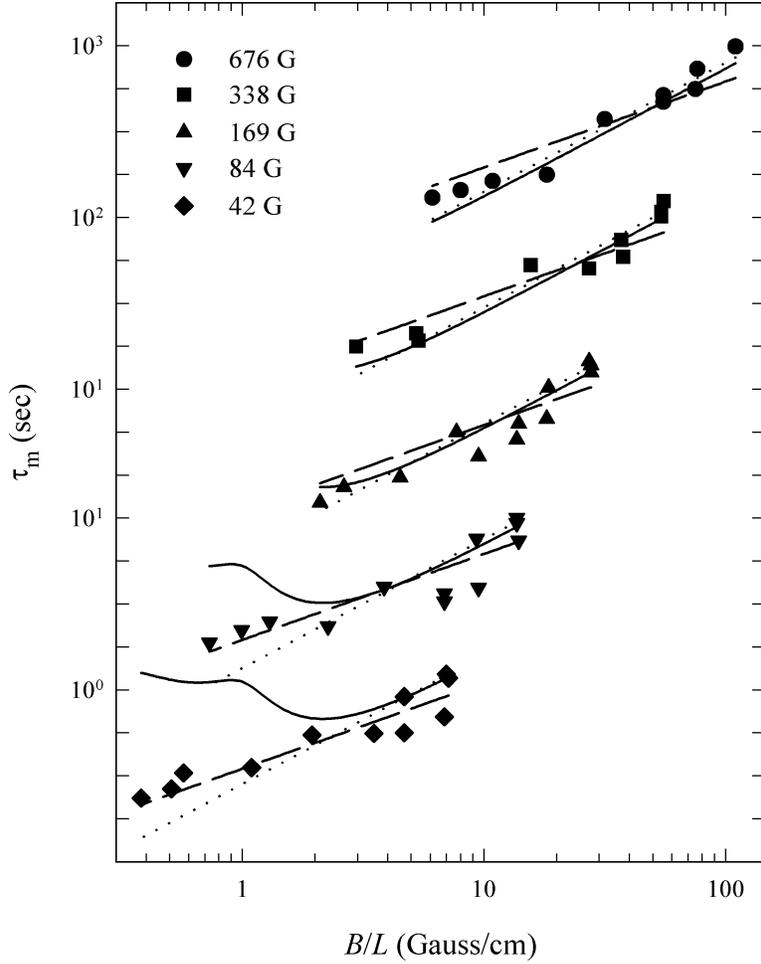
FIG. 4. Lifetime data from the V trap compared to quadrupole scaling with $B_{\mathrm{q}} \propto B$ (solid line), B^2/L^3 scaling (dotted line), and B^2/L^2 scaling (dashed line). The difference between the quadrupole scaling and simple B^2/L^3 scaling comes from the sum over the resonances in Eq. (4). The symbols are the measured lifetimes at the indicated **B** field. The vertical axis is logarithmic; each tick mark indicates one decade. The vertical scale for each **B** data set overlaps with its neighbors; the scale labels correspond to the approximate maximum lifetime for each **B**.